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Optimal Unemployment Insurance in General Equilibrium

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Abstract

[TO BE COMPLETED]

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1 Introduction

The empirical literature documents a substantial degree of labor income risk, with a large fraction actually caused by (un)employment risk.[1] The existing (un)employment risk points to inherently missing insurance markets due to informational frictions. Specifically, if the job seekers' effort choices are not publicly observable, benefit payments cannot be made contingent on the job search decision, and the existing moral hazard friction leads to a collapse of private unemployment insurance markets.[2] In designing the unemployment insurance system, the government has to weigh labor income insurance insurance against distorting the search incentives.[3]

In this paper, we use a dynamic general equilibrium search model of the labor market to compute an optimal unemployment insurance scheme when search effort choices are only private information of the households. In contrast to the existing literature on optimal unemployment insurance, we first provide a macroeconomic (general equilibrium) perspective, second allow for precautionary saving, and third preserve the analytical tractability in the sense that the optimal unemployment insurance system can be characterized without solving for the complete underlying wealth distribution. The first property is motivated by Lentz (2009) who find that the welfare effects in his labor market search model substantially depend on the relation between the time discount factor and the interest rate. In general equilibrium, the interest rate is endogenously determined such that we get rid of this additional degree of freedom. The second property is motivated by Shimer and Werning (2006, 2007, 2008) who show that when households make consumption-saving decisions, the optimal benefit profile differs substantially from the case in which households do not make the decision, as, e.g., in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). The third property allows us to consider richer unemployment duration dependent policies as in the previous literature. The

[1]For the United States, Hubbard, Skinner, and Zeldes (1995), Meghir and Pistaferri (2004), and Storesletten, Telmer, and Yaron (2004) estimate a standard deviation of the log of labor income for between 0.15 and 0.20. Furthermore, Jacobson, LaLonde, and Sullivan (1993) find that the long run earning loss upon job displacement is around 25 percent. Although Farber (2003) only finds losses half as large, it is undeniable, that job displacement and the associated unemployment spell substantially contribute to labor income risk. For a more detailed discussion, see Kletzer (1998).

[2]Arrow (1963) and Rothschild and Stiglitz (1976) have already made this point in different contexts.

[3]For an analysis of the trade-off between insurance and distorted incentives, see, e.g., Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Shimer and Werning (2006, 2007, 2008), and Pavoni (2007).

reason for this is that we do not have to compute the underlying wealth distribution as Hansen and Imrohoroglu (1992), Wang and Williamson (1996), and Young (2004). The combination of all three properties – which have quantitatively and computationally important implications – is unique to the literature.

The model we use is based on chapter Krebs and Scheffel (2010). Households are risk-avers and make consumption-saving, portfolio composition, and search effort decisions, the latter being unobservable to the government which causes the moral hazard friction. Despite the ex-post heterogeneity due to different employment histories, the equilibrium allocation is tractable in the sense that the equilibrium allocation can be characterized without knowing the complete underlying endogenous wealth distribution. This property is preserved in the optimal unemployment insurance system that is obtained by a mixed social planner – Ramsey problem. Specifically, the government chooses wealth independent transfer rates subject to the households' consumption-saving and search effort decisions, which is the Ramsey part of government's optimization problem, and the portfolio allocation, which is the social planner part of the government's optimization problem.

The main results are as follows: First, conditional on being employed, the social planner provides full insurance. This is due to the fact that there are no moral hazard frictions for currently employed households. Second, the optimal unemployment benefit rate is independent of the unemployment duration. This result is consistent with Shimer and Werning (2006, 2007, 2008) who show that under the absence of wealth effects, when there is a consumption saving decision, the benefit profile is constant with respect to unemployment duration. In contrast, Hopenhayn and Nicolini (1997) find falling benefit profiles without an endogenous consumption-saving decision. Third, while the net benefit rate for unemployed households is quite low, there are high rewards for successful job finders of 134 percent of their labor income. This result is consistent with Wang and Williamson (1996) and Hopenhayn and Nicolini (1997), but we find even stronger effects.

The rest of this paper is organized as follows: section 2 presents the model economy and section 3 constructs the competitive equilibrium. Section 4 sets up the restricted social planner problem. In section 5 based on a calibrated version of the model economy, we present some numerical results. Chapter 6 concludes.

2 Economy

The economy is populated by a continuum of *ex-ante* identical, infinitely-lived households with unit mass who derive period utility from consumption c_t and avoiding to exert job search effort l_t. For convenience we use lower case letters to denote idiosyncratic variables. The period utility function is given by $u(c_t, l_t) = \ln c_t - d(l_t)$, with $d(l_t)$ denoting the disutility from exerting job search effort. We assume that the utility cost of job search are increasing and convex in the exerted search effort, $d'(l_t) > 0$ and $d''(l_t) > 0$, and there is no search independent disutility of being unemployed, $d(0) = 0$. For a similar specification see, e.g., Lentz (2009).

Let $s_t \in S = \{e, u_1, u_2, u_3, \dots\}$ denote the current employment state of an arbitrary household. Households are either employed, $s_t = e$, or unemployed, $s_t = u_j$, where $j = 1, 2, 3, \dots$ denotes the duration of the current unemployment spell. The employment history of an arbitrary household is denoted by $s^t = (s_t, s_{t-1}, s_{t-2}, \dots)$. Let $\pi(s^t)$ denote the unconditional probability of experiencing employment history s^t. According to our normalization, the unconditional probability equals the mass of households with the respective employment history. The individual employment state follows a first-order Markov process with $\pi(s_{t+1}|s_t)$ denoting the probability of ending up in state s_{t+1} in the next period, given the household is currently in state s_t. While the probability of losing a job $\pi(u_1|e)$ is exogenously given, households determine their reemployment probability $\pi(e|u_j)$ by their search effort choice. For convenience, we leave the dependence of the state transition rate on search effort implicit. Clearly, the more search effort the household exerts, the higher the probability of finding a new job in the subsequent period, $(\partial \pi'(e|u_j))/(\partial l_t) > 0$, for $j = 1, 2, 3, \dots$.

Households hold physical and human capital, k_t and h_t from which they receive capital income $r_{kt}\ k_t$ and, if they are employed, labor income $r_{ht}(e)\ h_t$. There is no home production, $r_{ht}(u_j) = 0$. In addition, households receive (positive or negative) transfer payments proportional to their stock of human capital and dependent on their recent state transition. Specifically, the transfer payments are given by $Tr_t(s_t, s_{t-1})\ h_t$. Disposable income can be used for consumption and for investment into physical and human capital, x_{kt} and x_{ht}. Households maximize their lifetime utility with respect to consumption, investment in physical and human capital, and search effort. The optimization problem is given by

$$\max_{\{c_t, x_{kt}, x_{ht}, l_t\}} \left\{ U(\{c_t, x_{kt}, x_{ht}, l_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t (\ln c_t - d(l_t))\ \pi(s^t) \right\}$$

subject to

$$\begin{aligned} c_t + x_{kt} + x_{ht} &= r_{kt}\ k_t + r_{ht}(s_t)\ h_t + Tr_t(s_t, s_{t-1})\ h_t \\ k_{t+1} &= (1 - \delta_k)\ k_t + x_{kt} \\ h_{t+1} &= (1 - \delta_h(s_t))\ h_t + x_{ht} \\ k_{t+1} &\geq 0 \\ h_{t+1} &\geq 0 \end{aligned}$$

with δ_k and $\delta_h(s_t)$ denoting the (potentially state dependent) depreciation rate for physical and human capital.

There is a continuum of identical firms that produce the *all-purpose* good using physical and human capital as input factors. The production technology exhibits constant returns to scale. Hence, under competitive markets, the production sector can be represented by an aggregate firm with aggregate production technology $F(K_t, H_t^e)$. K_t denotes the aggregate stock of physical capital and H_t^e denotes the aggregate stock of human capital that is used in production. The profit maximization problem of the firm reads

$$\max_{K_t, H_t^e} \{\Pi(K_t,\ H_t^e) = F(K_t,\ H_t^e) - r_{kt}\ K_t - r_{ht}(e)\ H_t^e\}$$

The government sets transfer rates $Tr_t(s_t, s_{t-1})$ conditional on the households' recent state transition, and it has to keep a balanced budget in each period. The per period government budget constraint is given by

$$\sum_{s^t} Tr_t(s_t, s_{t-1})\ h_t(s^t)\ \pi(s^t) = 0$$

3 Competitive Equilibrium

Following Krebs (2003), we rewrite the households' optimization problem as a portfolio choice problem. Define total (nonhuman and human) wealth $w_t \doteq k_t + h_t$ and the share of physical capital with respect to wealth as $\theta_t \doteq \frac{k_t}{w_t}$. The return to total wealth can thus be written as

$$\begin{aligned} & r_t(\theta_t, s_t, s_{t-1}; r_{kt}, r_{ht}, Tr_t(s_t, s_{t-1})) \\ & \qquad = \theta_t \ (r_{kt} - \delta_k) + (1-\theta_t) \ (r_{ht}(s_t) - \delta_h) + (1-\theta_t) \ Tr_t(s_t, s_{t-1}), \quad \forall s_t, s_{t-1} \end{aligned}$$

and the constraints of the households' optimization problem simplify to

$$\begin{aligned} w_{t+1} &= (1 + r_t(\theta_t, s_t, s_{t-1}; r_{kt}, r_{ht}, Tr_t(s_t, s_{t-1}))) \ w_t - c_t \\ 0 &\leq \theta_{t+1} \leq 1 \end{aligned}$$

Instead of $\{c_t, x_{k,t+1}, x_{h,t+1}, l_t\}_{t=0}^{\infty}$, households now choose $\{c_t, w_{t+1}, \theta_{t+1}, l_t\}_{t=0}^{\infty}$ subject to the flow budget constraint and the portfolio share constraint (short-selling constraint).

We define a competitive equilibrium of this economy as follows:

Definition 1 (Competitive Equilibrium)**.**
A competitive equilibrium is

1. *A sequence* $\{K_t, \ H_t^e\}_{t=0}^{\infty}$ *that maximizes the firm's profit for a given sequence of factor prices* $\{r_{kt}, \ r_{ht}\}_{t=0}^{\infty}$*;*

2. *A sequence* $\{c_t, \ \theta_{t+1}, \ w_{t+1}, \ l_t\}_{t=0}^{\infty}$ *that solves the households' optimization problem for a given sequence of factor prices* $\{r_{kt}, \ r_{ht}\}_{t=0}^{\infty}$*, employment shocks* $\{s_t\}_{t=0}^{\infty}$*, and transfer payments* $\{Tr_t\}_{t=0}^{\infty}$*;*

3. *A sequence* $\{r_{kt}, \ r_{ht}\}_{t=0}^{\infty}$ *that satisfies market clearing on the input factor market,* $K_t = \sum_{s^t} \theta_t \ w_t(s^t) \ \pi(s^t)$ *and* $H_t^e = \sum_{s^{t-1}} (1-\theta_t) \ w_t(e, s^{t-1}) \ \pi(e, s^{t-1})$*, for all* t*;*

4. *A sequence of transfer payments* $\{Tr_t\}_{t=0}^{\infty}$ *that satisfies the per period government budget constraint for given saving policy and portfolio choice of all households.*

For the firm's optimization problem, the usual first-order conditions apply. Define the aggregate capital-to-labor ratio $\tilde{K}_t \doteq \frac{K_t}{H_t^e}$ and the production technology in intensive form $\tilde{F}(\tilde{K}_t) \doteq \frac{F(K_t, H_t^e)}{H_t^e}$. The profit maximization conditions are

$$
\begin{aligned}
r_{kt} &= \tilde{F}'(\tilde{K}_t) \\
r_{ht} &= \tilde{F}(\tilde{K}_t) - \tilde{K}_t \; \tilde{F}'(\tilde{K}_t)
\end{aligned}
$$

Because in equilibrium, factor prices are completely determined by the current aggregate capital-to-labor ratio, we can rewrite the returns to individual wealth as

$$
r_t(\theta_t, s_t, s_{t-1}; r_{kt}, r_{ht}, Tr_t(s_t, s_{t-1})) = r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1}))
$$

The individual state space of an arbitrary household consists of his capital share θ_t, current wealth w_t, and his recent state transition (s_t, s_{t-1}) which determines the transfer payments to be received. The aggregate state consists of the joint distribution of physical capital, human capital, and the employment state, on the one hand, and of the transfer system, on the other. For convenience, we leave the dependence of the value function from the aggregate state implicit. Rewriting the households' optimization problem in recursive form yields

$$
\tilde{V}(\theta_t, w_t, s_t, s_{t-1}) = \max_{c_t, w_{t+1}, \theta_{t+1}, l_t} \left\{ \ln c_t - d(l_t) + \beta \; \mathbb{E}[\tilde{V}(\theta_{t+1}, w_{t+1}, s_{t+1}, s_t)] \right\}
$$

subject to

$$
\begin{aligned}
w_{t+1} &= (1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1}))) \; w_t - c_t \\
0 &\leq \theta_{t+1} \leq 1
\end{aligned}
$$

Substituting for consumption using the flow budget constraint, the households' first-order conditions with respect to w_{t+1}, θ_{t+1}, and l_t read

$$\frac{1}{c_t} = \beta\ \mathbb{E}\left[\frac{1+(\theta_{t+1}, s_{t+1}, s_t; \tilde{K}_{t+1}, Tr_{t+1}(s_{t+1}, s_t))}{c_{t+1}}\right] \tag{1}$$

$$0 = \mathbb{E}\left[\frac{(r_{k,t+1}-\delta_k)-(r_{h,t+1}+Tr_{t+1}(s_{t+1}, s_t)-\delta_h(s_{t+1}))}{c_{t+1}}\right] \tag{2}$$

$$\frac{\partial d(l_t)}{\partial l_t} = \beta\ \frac{\partial \pi(e|u_j)}{\partial l_t}\ \left(\tilde{V}(\theta_{t+1}, w_{t+1}, e, u_j) - \tilde{V}(\theta_{t+1}, w_{t+1}, u_{j+1}, u_j)\right), \quad \forall j \tag{3}$$

A well established result states,that under linear homogeneity of disposable income in current wealth and homothetic preferences, the consumption and saving policies are also linear homogenous in wealth.[4] Specifically, it is easy to verify that the policies

$$c_t = (1-\beta)\ (1+r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ w_t \tag{4}$$

$$w_{t+1} = \beta\ (1+r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ w_t \tag{5}$$

solve the consumption-saving Euler equation (1) and the flow budget constraint. Inspection of (2) reveals that the portfolio choice does not exhibit any direct dependence on wealth if the consumption policy is linear. However, it is still possible that the portfolio choice depends indirectly on individual wealth holdings through wealth dependence of the search effort choice. By the method of guess and verify,[5] we show that

$$\tilde{V}(\theta_t, w_t, s_t, s_{t-1}) = \frac{\ln w_t}{1-\beta} + V(\theta_t, s_t, s_{t-1}) \tag{6}$$

where $V(\theta_t, s_t, s_{t-1})$ solves the intensive form Bellman equation

$$\begin{aligned} V(\theta_t, s_t, s_{t-1}) = \max_{l_t, \theta_{t+1}} \Bigg\{ & \ln(1-\beta) + \frac{\beta}{1-\beta}\ln\beta - d(l_t) \\ & + \frac{\ln(1+r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))}{1-\beta} + \beta\mathbb{E}\left[V(\theta_{t+1}, s_{t+1}, s_t)\right]\Bigg\} \end{aligned} \tag{7}$$

Clearly, using (6), the first-order condition with respect to search effort (3) becomes wealth independent as long as the portfolio choice is wealth independent. Thus, portfolio choices and search effort decisions are independent of the individual wealth holdings, but

[4]See, e.g., Krebs (2003) and Stokey (2009).

[5]For more details on the derivation of the value function based on the method of guess and verify, see appendix 6.

dependent on the current employment state, $\theta_{t+1} = \theta_{t+1}(s_t)$.

Having solved for the firm's and the households' policies, we now close the model by analyzing market clearing on the input factor markets. Define $\rho_t(s_{t-1}) \doteq \frac{\sum_{s^{t-2}} w_t(s^{t-1})\pi(s^{t-1})}{\sum_{s^{t-1}} w_t(s^{t-1})\pi(s^{t-1})}$ as the relative wealth owned by all households whose employment state was s_{t-1} in the preceding period. Using the definition of the portfolio shares and the saving policy, the law of motion of the wealth measure is given by[6]

$$\rho_{t+1}(s_t) = \frac{\sum_{s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ \rho_t(s_{t-1})}{\sum_{s_t, s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))\ \rho_t(s_{t-1})} \tag{8}$$

which yields the following market clearing condition

$$\tilde{K}_{t+1} = \frac{\sum_{s_t} \theta_{t+1}(s_t)\ \rho_{t+1}(s_t)}{\sum_{s_t}(1 - \theta_{t+1}(s_t))\ \pi(e|s_t)\ \rho_{t+1}(s_t)} \tag{9}$$

We summarize the equilibrium characterization in the following proposition:

Proposition 1 (Characterization of Competitive Equilibrium)**.**
For any transfer scheme that satisfies the per period budget constraint of the government

$$\sum_{s_t, s_{t-1}} Tr_t(s_t, s_{t-1})\ \pi(s_t|s_{t-1})\ \rho_t(s_{t-1}) = 0$$

a competitive equilibrium can be characterized as follows:

1. *The firm's demand for physical and human capital satisfies the usual first-order conditions of profit maximization under competitive markets;*

2. *The households consumption and savings policies are linear homogenous in wealth and given by* (4) *and* (5)*, the portfolio choices and search effort decisions are wealth independent and implicitly given as the solution of* (2) *and* (3) *where the intensive form value function solves the respective intensive form Bellman equation* (7)*;*

3. *Market clearing satisfies* (9) *with the evolution of wealth shares governed by* (8)*.*

Observe that the equilibrium allocation is independent of the unconditional wealth distribution. Specifically, it suffices to solve for the relative wealth owned by households of

[6] Form more details on the derivation of the law of motion of the wealth shares, see appendix 6.

type s_{t-1}. Clearly, the wealth ratios are a substantially easier mathematical object than the unconditional wealth distribution.

4 Optimal Unemployment Insurance

In a frictionless environment, the social planner would choose an allocation that provides full insurance against income shocks. However, for the social planner, the exerted job search effort of the individual households' is unobservable. Providing too generous insurance against unemployment restrain households from exerting sufficient search effort. Optimal unemployment insurance thus has to take the existing moral hazard friction into account.

For the analysis here, we restrict to a very specific social planner problem. The social planner's objective function, the social welfare function, is derived under two assumptions: First, the social planner weighs the lifetime utility of the individual households equally. Hence, we rule out transfer payments across types of households solely based on the fact that the social planner cares more about one type of agents than of than other. Second, although we allow the social planner to choose the consumption-saving decision of the households, we restrict to allocations that are self-enforcing in the sense that the households' consumption-saving Euler equations have to be satisfied. This assumption guarantees that the linear homogenous consumption and saving policies derived in the previous section still hold. Hence, the decomposition of the value function into an intensive form value function on the one hand, and a wealth dependent term, on the other, is still valid. The social welfare function can now be written as

$$\int_I \tilde{V}(\theta_t, w_t, s_t, s_{t-1})\mathrm{d}i = \sum_{s_t, s_{t-1}} \mu_t(s_t, s_{t-1}) V(\theta_t, s_t, s_{t-1}) + \int_I \frac{\ln w_t}{1-\beta}\mathrm{d}i$$

where $\mu(s_t, s_{t-1})$ denotes the mass of households of type (s_t, s_{t-1}). Clearly, with w_t being a state variable to the planner's problem in period t, it is sufficient for the social planner to confine attention to the social welfare function in intensive form, which is defined as the first term on the right hand side of the equation.

We restrict our analysis to the case where a social planner can fully commit to his plans. Clearly, this means that the current transfer scheme was determined in the previous period and is thus a state variable to the social planner's problem in the current period. Hence, the social planner cannot announce a policy in the current period that induces the households to exert high search effort and switch to a different high insurance policy after the search effort decision materializes into low unemployment rates in the subsequent period. This assumption has important implications on the optimal unemployment insurance system we derive. As shown by Krusell, Quadrini, and Rios-Rull (1997), Krusell (2002) for redistributive policies in general and Kankanamge and Weitzenblum (2011) for unemployment insurance specifically, implementing a time consistent insurance system comes at high welfare costs compared to the case in which the social planner can fully commit. Specifically, under limited commitment, there is more unemployment insurance at the cost of substantially higher unemployment. The optimal unemployment insurance system we derive is thus under the absence of this implementability friction.

Because search effort is unobservable for the social planner, the households' first-order conditions with respect to search effort enter as an additional constraint to the social planner's optimization problem. With individual value functions entering the constraints, standard dynamic programming techniques are not valid.[7] The standard way of dealing with this problem is to define a broader state space that also includes the current lifetime utility of the individual households. As shown by Spear and Srivastava (1987), the social planner problem is recursive on this new state space, making standard dynamic programming techniques applicable. However, this method comes at the cost that not every solution to the first-order conditions of this augmented problem is a solution to the original planner problem. Thus, we have to check whether the promised utility of the candidate solution solves the households' functional equation when the consumption and saving policies as well as the portfolio choices are given by the candidate solution.

For the incentive problem considered here, it suffices to consider the difference in lifetime utility. Thus, instead of lifetime utility, we only include the difference of lifetime utilities as state variable, reducing the dimension of the state space. Define $\Delta V_t(u_j, s_{t-1}) \doteq V(\theta_t, e, s_{t-1}) - V_t(\theta_t, u_j, s_{t-1}), j = 1, 2, 3, \ldots$ as the utility difference.

[7] See, e.g. Abraham and Pavoni (2008), Mele (2010), and Marcet and Marimon (2011).

From the households' utility functions, we get

$$\begin{aligned}
\Delta V_t(u_j, s_{t-1})) = d(l(u_j)) &+ \frac{\ln(1 + r_t(\theta_t, e, s_{t-1}; \tilde{K}_t, Tr_t(e, s_{t-1})))}{1-\beta} \\
&- \frac{\ln(1 + r_t(\theta_t, u_j, s_{t-1}; \tilde{K}_t, Tr_t(u_j, s_{t-1})))}{1-\beta} \\
&- \beta \left[\pi(u_1|e)\Delta V_{t+1}(u_1, e) - \pi(u_{j+1}|u_j)\Delta V_{t+1}(u_{j+1}, u_j)\right] \\
&+ \beta \, \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, e; \tilde{K}_{t+1}, Tr_{t+1}(e, e)))}{1-\beta} \\
&- \beta \, \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, u_j; \tilde{K}_{t+1}, Tr_{t+1}(e, u_j)))}{1-\beta}, \forall j = 1, 2,
\end{aligned}$$

This condition is the promise keeping constraint since it requires the government to deliver the utility difference $\Delta V_t(u_j, s_{t-1})$ that was promised in the previous period.

Let $W(\theta_t, k_t, \rho_t, \mu_t, Tr_t, \Delta V_t)$ denote the social welfare. The social planner chooses the portfolio share $\theta_{t+1}(s_t)$, the search effort $l_t(s_t)$, the capital-to-labor ratio $\tilde{K}_t$, the wealth shares $\rho_{t+1}(s_t)$, the distribution of households across states $\mu_{t+1}(s_{t+1}, s_t)$, the transition dependent transfer payments $Tr_{t+1}(s_{t+1}, s_t)$, and the promised utility difference $\Delta V_{t+1}(u_{j+1}, s_t)$ for all household types in order to maximize

$$\begin{aligned}
W_t(\theta_t, \tilde{K}_t, \rho_t, \mu_t, Tr_t, \Delta V_t) = \max_{\theta_{t+1}, l_t, \tilde{K}_{t+1}, \rho_{t+1}, \mu_{t+1}, Tr_{t+1}, \Delta V_{t+1}} \Bigg\{ B - \sum_{j, s_{t-1}} \mu(u_j, s_{t-1}) \, d(l) \\
+ \sum_{s_t, s_{t-1}} \mu(s_t, s_{t-1}) \, \frac{\ln(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1})))}{1-\beta} \\
+ \beta \, W_{t+1}(\theta_{t+1}, \tilde{K}_{t+1}, \rho_{t+1}, \mu_{t+1}, Tr_{t+1}, \Delta V_{t+1}) \Bigg\}
\end{aligned}$$

subject to

i.) the promise keeping constraint

$$
\begin{aligned}
\Delta V_t(u_j, s_{t-1})) = d(l(u_j)) &+ \frac{\ln(1 + r_t(\theta_t, e, s_{t-1}; \tilde{K}_t, Tr_t(e, s_{t-1})))}{1-\beta} \\
&- \frac{\ln(1 + r_t(\theta_t, u_j, s_{t-1}; \tilde{K}_t, Tr_t(u_j, s_{t-1})))}{1-\beta} \\
&- \beta \left[\pi(u_1|e)\Delta V_{t+1}(u_1, e) - \pi(u_{j+1}|u_j)\Delta V_{t+1}(u_{j+1}, u_j) \right] \\
&+ \beta \, \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, e; \tilde{K}_{t+1}, Tr_{t+1}(e, e)))}{1-\beta} \\
&- \beta \, \frac{\ln(1 + r_{t+1}(\theta_{t+1}, e, u_j; \tilde{K}_{t+1}, Tr_{t+1}(e, u_j)))}{1-\beta}, \forall j = 1, 2, \ldots
\end{aligned}
$$

ii.) the incentive compatibility constraint

$$
\frac{\partial d(l_t)}{\partial l_t} = \beta \, \frac{\partial \pi(e|u_j)}{\partial l_t} \, \Delta V_{t+1}(u_{j+1}, u_j)
$$

iii.)the evolution of wealth shares

$$
\rho_{t+1}(s_t) = \frac{\sum_{s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1}))) \; \rho_t(s_{t-1})}{\sum_{s_t, s_{t-1}} \pi(s_t|s_{t-1})(1 + r_t(\theta_t, s_t, s_{t-1}; \tilde{K}_t, Tr_t(s_t, s_{t-1}))) \; \rho_t(s_{t-1})}
$$

iv.)the market clearing condition

$$
\tilde{K}_{t+1} = \frac{\sum_{s_t} \theta_{t+1}(s_t) \; \rho_{t+1}(s_t)}{\sum_{s_t} (1 - \theta_{t+1}(s_t)) \; \pi(e|s_t) \; \rho_{t+1}(s_t)}
$$

v.)the government budget constraint

$$
0 = \sum_{s_{t+1}, s_t} Tr_{t+1}(s_{t+1}, s_t) \; \pi(s_{t+1}|s_t) \; \rho_{t+1}(s_t)
$$

vi.)the evolution of population shares

$$
\mu_{t+1}(s_{t+1}, s_t) = \sum_{s_{t-1}} \pi(s_{t+1}|s_t) \; \mu_t(s_t, s_{t-1})
$$

5 Quantitative Analysis

5.1 Specification and Calibration

The quantitative analysis here uses two additional restrictions. First, we impose an *ad hoc* irreversibility constraint on human capital, which restricts the social planner in his portfolio choices and transfer policies for unemployed households. Specifically, for unemployed households the choices have to satisfy $(1-\theta_{t+1})\ w_{t+1} \geq (1-\delta_h(s_t))\ (1-\theta_t)\ w_t$. With saving policies that are linear homogenous in wealth, this condition is wealth independent as well. Second, we focus on the long-run (stationary) optimal unemployment insurance system, although we are aware that the welfare gains may be offset by costly transition phases, as demonstrated by Gilles and Weitzenblum (2003).

The model is calibrated on a monthly basis to match stylized facts of the US economy. In particular, we calibrate to match the elasticity of the job finding rate with respect to benefit payments, the unemployment rate, and the monthly equilibrium growth rate. This approach is motivated by two observations: First, the elasticity takes central stage in determining the employment effects of changes in the unemployment benefit system, as shown in the sensitivity analysis of in Krebs and Scheffel (2010). Second, the growth rate is key for the determination of the welfare effect. Specifically, the welfare effect is mainly determined by the consumption volatility of the employed households and its magnitude is directly linked to the the average consumption growth.

The functional forms are specified as follows: the production technology is of the Cobb-Douglas type, $F(K_t, H_t^e) = z\ K_t^{\alpha}\ H_t^{e(1-\alpha)}$, the disutility of search is a power function $d(l_t) = A\ l_t^{\zeta}$, and the job search technology is an exponential function $\pi(e|s_t) = 1 - \mathrm{e}^{-\lambda l_t}$.[8] We set the capital share in production α to 0.36, and the depreciation rates of physical and human capital δ_k and $\delta_h(s_t)$ to 0.0050 which amounts to six percent per annum. The depreciation rate of physical capital is within the range suggested by the literature. In contrast, the individual depreciation rate of human capital is estimated to be between zero and four percent per annum.[9] However, our infinite horizon model has to account for the additional mortality based human capital depreciation, which is not included in the estimates. Assuming a working life span of 50 years, we have to add an additional

[8] For the specification of the disutility function and the job search technology, see e.g. Lentz (2009).
[9] See Browning, Hansen, and Heckman (1999).

Table 1: Calibration - Exogenous Parameters

PARAMETER	DESCRIPTION	VALUE
PREFERENCES		
A	parameter of disutility of search	1.0000
β	time preference rate	0.9950
PRODUCTION		
α	capital share	0.3600
DEPRECIATION RATES		
δ_k	depreciation rate: physical capital	0.0050
$\delta_h(s_t)$	depreciation rate: human capital if employed	0.0050
LABOR MARKET AND TRANSITION RATES		
$\pi(u_1\|e)$	monthly job destruction rate	0.0300

depreciation of 2 percent per annum, which makes our chosen value for the depreciation rate to be at the upper end of the range. Following Shimer (2005), the monthly job separation rate $\pi(u_1|e)$ is three percent. The time preference factor β is set to 0.9950 which is 0.94 on an annual basis. Moreover, the scaling factor of disutility of search effort A is set to one. Actually this value is no restriction since it cannot be identified independently from the search technology parameter λ that will be used to match the unemployment rate of 8 percent. The curvature of the disutility function is set to five, which implies an equilibrium reemployment elasticity of -0.25, as found by Meyer and Mok (2007). Finally, setting the scaling parameter of the production technology to 0.0155 yields a monthly consumption growth rate of 0.2 percent, which amounts to approximately three percent per annum.

5.2 Results

There are three main results: First, in the absence of any moral hazard friction for employed households, the government provides full insurance conditional on being em-

Table 2: Calibration - Endogenous Parameters

PARAMETER	DESCRIPTION	VALUE
PREFERENCES		
ζ	curvature of disutility of search	5.0000
PRODUCTION		
z	productivity	0.0155
LABOR MARKET AND TRANSITION RATES		
λ	search technology parameter:	3.0000
PARAMETERS ARE CHOSEN TO MATCH		
	aggregate monthly consumption growth rate	0.0025
	unemployment rate	0.0750
	average benefit elasticity of reemployment probability	-0.2500

ployed. Specifically, the consumption growth rate is 0.27 percent, for sure. If households remain employed until the next period, the transfer payments are negative and can be interpreted as a labor income tax of 7.2 percent. In contrast, when becoming unemployed, households receive transfer payments that amount to 92.8 percent of their previous gross income.

Second, transfer rates to unemployed households that are unemployed for at least 2 periods are independent of the duration of the current unemployment spell. This result is due to the absence of wealth effects when preferences are homothetic and disposable income is linear homogenous in wealth. Our result is similar to Shimer and Werning (2006, 2007, 2008) who get rid of the wealth effects by using a CARA utility specification. However, the optimal de-trended consumption profile for unemployed households is nevertheless decreasing, because the unemployed's wealth position grows at a lower rate than the economy wide average wealth positions.

Third, for unemployed households, benefit payments are quite low at 5.5 percent of the gross wage (and 12.7 percent of net wages compared to households that are employed for at least two periods). However, if unemployed households are successful job seekers, they receive a reward as high as 134 percent of their gross wage. The government uses the spread between the low benefit rate for unemployed and the high reward for successful job seekers to provide sufficient incentives for the households to exert search effort. Although this result is not new to the literature, see e.g. Wang and Williamson (1996) and Hopenhayn and Nicolini (1997), we find a substantial higher spread, again mainly due to the absence of wealth effects.

Table 3: Results

EMPLOYMENT STATE	CONSUMPTION GROWTH RATE		IMPLIED BENEFIT RATE	
	$s_{t+1} = e$	$s_{t+1} = u_{j+1}$	$s_{t+1} = e$	$s_{t+1} = u_{j+1}$
$s_t = e$	+0.27%	+0.27%	−7.2%	+92.8%
$s_{t+1} = u_j$	+1.0%	−0.20%	+134%	+5.42%

6 Conclusions

We constructed a specific social planner problem that allowed us to compute an optimal unemployment insurance system in a dynamic general equilibrium model with publicly unobservable job search effort by the households without solving for the within type wealth distribution. Our quantitative results are derived under the assumption that the government can fully commit to its policy plans. The main results are as follows: first, there is full insurance for currently employed households, second, the optimal profile of the benefit rate is independent of the duration of the current unemployment spell, and third, benefits are low for unemployed households but the reward for successful job search is substantial. The results are consistent with the existing literature on optimal unemployment insurance.

There are two directions for future research building on the model presented here. First, we plan to use the model to derive more theoretical results on optimal unemployment insurance, in particular concerning the importance of the general equilibrium effect. The wealth independence of the optimal unemployment insurance system may will simplify the analysis substantially. We will analyze the general equilibrium effect in a calibrated version of the model economy quantitatively. Second, we plan to extend the analysis to limited commitment of the government in the sense that it can deviate from its previously announced policies. Clearly, since the current transfer scheme has no impact on the households' allocation decisions, the government is tempted to provide more insurance in the current period. As argued by Kankanamge and Weitzenblum (2011), the time-consistent unemployment insurance system under limited commitment can be fundamentally different from the optimal unemployment insurance system under full commitment. This analysis will shed light on the question on how quantitatively important this additional friction is.

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Appendix: Guess and Verify of Value Function

For the derivation of the functional form of the value function, we leave the dependency of aggregate state variables for convenience implicit. Guess that the functional form is given by

$$\tilde{V}(\theta_t,\ w_t,\ s_t,\ s_{t-1}) = A(s_t,\ s_{t-1})\ \ln[(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t] + B(s_t,\ s_{t-1}) \tag{10}$$

Together with the optimal consumption policy, the Bellman equation reads

$$\begin{aligned} A(s_t,\ s_{t-1})\ \ln[(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t] + B(s_t,\ s_{t-1}) \\ = \max_{\theta_{t+1},l_t} \Big\{ \ln(1-\beta) + \ln[(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t] \\ - \mathbf{1}_{s_t=u_j}\ d(l_t) + \beta\ \mathbb{E}\Big[A(s_{t+1},\ s_t)\ln[(1+r(\theta_{t+1},\ s_{t+1},\ s_t))\ w_{t+1}] + B(s_{t+1},\ s_t)\Big]\Big\} \end{aligned}$$

Using the saving policy $w_{t+1} = \beta\ (1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t$, we rewrite the Bellman equation as

$$\begin{aligned} A(s_t,\ s_{t-1})\ \ln[(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t] + B(s_t,\ s_{t-1}) \\ = \max_{\theta_{t+1},l_t} \Big\{(1+\beta\ \mathbb{E}[A(s_{t+1},\ s_t)])\ \ln[(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t] + \ln(1-\beta) + \beta\ \mathbb{E}[A(s_{t+1},\ s_t)]\ \ln\beta \\ - \mathbf{1}_{s_t=u_j}\ d(l_t) + \beta\ \mathbb{E}\Big[A(s_{t+1},\ s_t)\ \ln(1+r(\theta_{t+1},\ s_{t+1},\ s_t)) + B(s_{t+1},\ s_t)\Big]\Big\} \end{aligned}$$

Now suppose, we already found a solution θ_{t+1} and l_t that are independent of current state variables θ_t and w_t. By the method of undetermined coefficients, we get

$$A(s_t,\ s_{t-1}) = \frac{1}{1-\beta}$$

and the Bellman equation then simplifies to

$$\begin{aligned} B(s_t,\ s_{t-1}) = \max_{\theta_{t+1},l_t} \Big\{ \ln(1-\beta) + \frac{\beta}{1-\beta}\ln\beta - \mathbf{1}_{s_t=u_j}\ d(l_{it}) \\ + \beta\ \mathbb{E}\Big[\frac{1}{1-\beta}\ln(1+r(\theta_{t+1},\ s_{t+1},\ s_t)) + B(s_{t+1},\ s_t)\Big]\Big\} \end{aligned} \tag{11}$$

Obviously, the optimal portfolio choice as well as the optimal search intensity that solve the Bellman equation in intensive form, equation (11) are independent of the current state variables θ_t and w_t. In other words: θ_{t+1} and l_t only depend on s_t and the exogenous model parameters, which is consistent with our previous conjecture. The optimal policies transform the functional equation (11) into the respective plan equation such that we can easily solve for $B(s_t,\ s_{t-1})$, thereby verifying our initial guess on the functional form of the value function, equation (10). This completes our proof.

Appendix: Evolution of Wealth Shares and Market Clearing

Let $s^t = (s_t, s_{t-1}, s_{t-2}, \dots)$ denote the history of shocks, which describes the history of type-s^t-agent. Let $W_{t+1}(s_t, s_{t-1})$ denote the wealth owned by households in $t+1$ who experienced a truncated employment history (s_t, s_{t-1}), and W_{t+1} denote the total (economy wide) wealth. Define the relative wealth share

$$\begin{aligned}\rho_{t+1}(s_t, s_{t-1}) &\doteq \frac{W_{t+1}(s_t, s_{t-1})}{W_{t+1}} \\ &= \frac{\pi(s_t|s_{t-1})\ \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1}|s^{t-2})\ \pi(s^{t-2})}{\sum_{s^t} w_{t+1}(s^t)\ \pi(s^t)}\end{aligned}$$

where we suppressed the dependency of the conditional probabilities from individual search effort decisions, for convenience. By the households' saving policies $w_{t+1} = \beta\,(1+ r(\theta_t,\ s_t,\ s_{t-1}))\ w_t$,[10] the relative wealth share can be rewritten as

$$\begin{aligned}&\rho_{t+1}(s_t, s_{t-1}) \\ =&\frac{\pi(s_t|s_{t-1})\ \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1}|s^{t-2})\ \pi(s^{t-2})}{\sum_{s^t} w_{t+1}(s^t)\ \pi(s^t)} \\ =&\frac{\pi(s_t|s_{t-1})\ \sum_{s^{t-2}}(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t(s^{t-1})\ \pi(s_{t-1}|s_{t-2})\ \pi(s_{t-2}|s^{t-3})\ \pi(s^{t-3})}{\sum_{s^t}(1+r(\theta_t,\ s_t,\ s_{t-1}))\ w_t(s^{t-1})\ \pi(s_t|s^{t-1})\ \pi(s^{t-1})} \\ =&\frac{\pi(s_t|s_{t-1})\ (1+r(\theta_t,\ s_t,\ s_{t-1}))\ \sum_{s_{t-2}} \pi(s_{t-1}|s_{t-2})\ \sum_{s^{t-3}} w_t(s^{t-1})\ \pi(s_{t-2}|s^{t-3})\ \pi(s^{t-3})}{\sum_{s_t}\sum_{s_{t-1}} \pi(s_t|s_{t-1})\ (1+r(\theta_t,\ s_t,\ s_{t-1}))\ \sum_{s^{t-2}} w_t(s^{t-1})\ \pi(s_{t-1}|s^{t-2})\ \pi(s^{t-2})} \\ =&\frac{\pi(s_t|s_{t-1})\ (1+r(\theta_t,\ s_t,\ s_{t-1}))\ \sum_{s_{t-2}} \rho_t(s_{t-1},\ s_{t-2})}{\sum_{s_t}\sum_{s_{t-1}} \pi(s_t|s_{t-1})\ (1+r(\theta_t,\ s_t,\ s_{t-1}))\ \sum_{s_{t-2}} \rho_t(s_{t-1},\ s_{t-2})} \qquad (12)\end{aligned}$$

where the last line follows from applying the definition of the wealth share once more. Note that equation (12) is the equilibrium law of motion of the wealth shares.

Now, consider the aggregate stock of physical and human capital used in production.

[10] For convenience, we leave the dependency of the return function on the aggregate state implicit.

By definition, the stock of physical capital used in production reads

$$\begin{aligned}
K_{t+1} &= \sum_{s^t} k_{t+1}(s^t)\ \pi(s^t) = \sum_{s^t} \theta_{t+1}(s_t)\ w_{t+1}(s^t)\ \pi(s^t) = \sum_{s^t} \theta_{t+1}(s^t)\ w_{t+1}(s^t)\ \pi(s^t) \\
&= \sum_{s_t} \sum_{s_{t-1}} \sum_{s^{t-2}} \theta_{t+1}(s_t)\ w_{t+1}(s^t)\ \pi(s_t|s_{t-1})\ \pi(s_{t-1}|s^{t-2})\ \pi(s^{t-2}) \\
&= \sum_{s_t} \sum_{s_{t-1}} \theta_{t+1}(s_t)\ \pi(s_t|s_{t-1}) \sum_{s^{t-2}} w_{t+1}(s^t)\ \pi(s_{t-1}|s^{t-2})\ \pi(s^{t-2}) \\
&= \sum_{s_t} \sum_{s_{t-1}} \theta_{t+1}(s_t)\ W_{t+1}(s_t, s_{t-1}) \\
&= \sum_{s_t} \sum_{s_{t-1}} \theta_{t+1}(s_t)\ \rho_{t+1}(s_t, s_{t-1})\ W_{t+1} \qquad (13)
\end{aligned}$$

For the stock of human capital that is used in production, only those agents that are currently employed, are relevant. Hence,

$$H^e_{t+1} = \sum_{s^t} \pi(s_{1,t+1} = e|s^t)\ (1 - \theta_{t+1}(s_t))\ w_{t+1}(s^t)\ \pi(s^t)$$

which simplifies, by the similar procedure as above, to

$$H^e_{t+1} = \sum_{s_t} \sum_{s_{t-1}} \pi(s_{1,t+1} = e|s_t)\ (1 - \theta_{t+1}(s_t))\ \rho_{t+1}(s_t, s_{t-1})\ W_{t+1} \qquad (14)$$

Taken together, (13) and (14) jointly determine the capital to labor ratio

$$\tilde{K}_{t+1} = \frac{\sum_{s_t} \sum_{s_{t-1}} \theta_{t+1}(s_t)\ \rho_{t+1}(s_t, s_{t-1})}{\sum_{s_t} \sum_{s_{t-1}} \pi(s_{t+1} = e|s_t)\ (1 - \theta_{t+1}(s_t))\ \rho_{t+1}(s_t, s_{t-1})} \qquad (15)$$

Clearly, stationarity of the equilibrium implies $\rho_t = \rho$, $\forall t$. Thus, every $\tilde{K}$ that solves this condition (note that the ratios ρ defined previously, depend on the capital to labor ratio via the return functions) implicitly solves market clearing on the input factor markets. moreover, market clearing is independent of aggregate wealth.